Living Together: Planets, Host Stars, and Binaries

Eclipsing binary systems as tests of low−mass stellar evolution theory

Gregory A. Feiden[1]

[1] Uppsala University, Uppsala, Sweden, gregory.feiden@physics.uu.se

Abstract

Stellar fundamental properties—masses, radii, effective temperatures—can be extracted from observations of eclipsing binary systems with remarkable precision, often better than 2%. Such precise measurements afford us the opportunity to confront the validity of basic predictions of stellar evolution theory, such as the mass–radius relationship. A brief historical overview of confrontations between stellar models and data from eclipsing binaries is given, highlighting key results and physical insight that have led directly to our present understanding. The current paradigm that standard stellar evolution theory is insufficient to describe the most basic relation, that of a star's mass to its radius, along the main sequence is then described. Departures of theoretical expectations from empirical data, however, provide a rich opportunity to explore various physical solutions, improving our understanding of important stellar astrophysical processes.

1 Introduction

Eclipsing binary (EB) systems are some of the best tools for testing stellar evolution theory. Advancements in stellar evolution theory motivated by confrontations between stellar model predictions and observations of EBs are second, perhaps, only to studies of globular cluster color-magnitude diagrams. The power of EB observations to further stellar evolution theory is derived from observer's ability to extract accurate stellar masses and radii of the individual components. Being that mass is the primary input parameter in stellar models and radius is a primary output quantity, EBs permit direct comparison with model predictions at the most fundamental level. Furthermore, the presence of two, presumably coeval, stars in a binary system forces stellar models to predict the properties of both stars at the same age, effectively making each binary system a miniature star cluster.

This review will attempt to characterize advances in low-mass stellar evolution theory that were and are motivated by investigations of EBs. Naturally, the definition of a low-mass star is strongly dependent on context, so for reference, I will refer to low-mass stars as stars with masses below 0.8 $M_\odot$. Stars in the range of 0.8 to 1.2 $M_\odot$ will be referred to as "solar-like" stars. These definitions are largely set by the characteristics of stellar evolution models in the given mass regimes. The lower boundary of 0.8 $M_\odot$ is defined by the growing importance of non-ideal contributions to the gas equation of state and a need for detailed radiative transfer models to prescribe the surface boundary condition. Additionally, models below the 0.8 $M_\odot$ boundary are less sensitive to model input parameters such as metallicity, the convective mixing length, and age compared to models above that threshold. The upper boundary to the "solar-like" regime at 1.2 $M_\odot$ corresponds, roughly, to the mass threshold above which main-sequence stars are believed to develop a convective core.

Definitions outlined in the previous paragraph are the result of knowledge accumulated through decades of research on stellar evolution. The importance of many of the above effects were revealed over time, some through the study of EBs. It is therefore instructive to begin by placing our modern understanding of low-mass stellar evolution theory into perspective with a recapitulation of historical progress. Following the historical evolution of stellar evolution theory, a summary is provided of where we are today, what is "state of the art," and where EBs are making significant contributions to the advancement of stellar evolution models. Finally, ideas about how EBs may contribute significantly to stellar evolution theory are presented.

Before embarking on the historical perspective, one must caution that the historical review presented below attempts to cover the most important advances in the state of low-mass stellar evolution theory. As such, individual studies contributing to the the knowledge and inspiration of those studies cited below may appear under appreciated, as might the effort that goes into measuring stellar masses and radii. There is no intent to minimize the meticulous work of theorists and observers who put tireless hours into advancing physical models and extracting exquisite measurements from their data. Without these efforts, theorists would be unable to construct advanced computational models and they would be unable to casually remark on the agreement between observations and theory.

2 Historical perspective

The history of testing low-mass stellar evolution theory with EBs begins with the first observations of YY Geminorum (hereafter YY Gem, also Castor C; Adams & Joy 1920) and its subsequent classification as an EB (van Gent 1926; Joy & Sanford 1926). YY Gem consists of two M0-type stars that are nearly identical when it comes to their fundamental properties ($M \approx 0.60$ $M_\odot$, $R \approx 0.62$ $R_\odot$, $T_{\rm eff} \approx$ 3800 K). Although it would be another 20 years before YY Gem provided clues about the inadequacy of stellar model physics, it will be continually mentioned throughout the historical development of low-mass stellar evolution theory.

The first major contribution provided by YY Gem (and EBs) to low-mass stellar evolution theory came in the early 1950s. At this point, significant advancements in energy generation processes via nuclear reactions had been made and the proton–proton (p–p) chain was established as the dominant energy production mechanism in late-type stars (Aller 1950; Salpeter 1952). However, comparisons of stellar evolution models computed assuming energy production via p–p chain predicted stellar luminosities far in excess of observed luminosities for stars below the Sun in the HR-diagram (Aller 1950; Aller et al. 1952). Strömgren (1952) identified numerous possible shortcomings in stellar model physics, including the validity of the equation of state (EOS) and adopted radiative opacities. Most importantly, Strömgren noted that the prevailing assumption of radiative equilibrium (e.g., Eddington 1926) throughout stellar interiors was likely incorrect and should be abandoned. Instead, it was argued that the properties of a surface hydrogen convection zone in late-type stars should be explored.

Following Strömgren's suggestion, Osterbrock (1953) computed the first set of stellar models of late-type stars with an extended outer convective envelope. Osterbrock found satisfactory agreement between model calculations with a deep convective envelope and observations of YY Gem for models where the hydrogen convection zone comprised the outer 30% of the star, by radius. The study by Osterbrock, motivated by observational properties of YY Gem, revealed that stellar models must account for the transport of energy by convection, a point that seems rather trivial from our modern point of view, but represented a significant advance in stellar evolution theory, at the time. However, while the inclusion of an outer convection zone provided agreement between model predictions and the properties of YY Gem, there was still significant disagreement between models and the properties of stars of later spectral type. Most notable were disagreements with the stars in Krüger 60, a visual binary consisting to two mid-to-late M-dwarfs (today classified as M2.0 and M4.0). Although the system is not eclipsing, its role in stellar structure theory is important and worth mentioning. Both stars were observed have a luminosity well below the predictions of stellar evolution models, much the same as the stars in YY Gem before the introduction of a hydrogen convection zone. These remaining errors were initially attributed to missing effects of electron conduction as an energy transport mechanism (Osterbrock 1953).

Carefully ruling out severe observational errors in the determination of stellar properties for Krüger 60, Limber (1958a) pointed out that several additional pieces of physics were likely necessary to reconcile model predictions with the observations. These physics included partial electron degeneracy of the stellar interior and a more rigorous treatment of radiative and convective temperature gradients for computation of energy transport in stellar interiors. However, even when all of these effects were included in models, significant disagreements with Krüger 60 remained (Limber 1958b). Instead, Limber extrapolated on the advances provided by Osterbrock (1953) and allowed the model interiors to be in full convective equilibrium. Under this assumption, models were able to provide predictions consistent with the properties of the stars in Krüger 60 (Limber 1958a,b).

Precise measurement of the fundamental properties of the stars in YY Gem helped initiate the establishment of the current paradigm that outer convection zones grow deeper toward later spectral types, effectively overturning the prevailing hypothesis of radiative equilibrium. The revolution was so complete that stellar interiors of the latest type stars were determined to be fully convective! However, agreement between models and data was foreseen to be short-lived. Observational data still possessed large error bars that greatly helped to mask the identification of further disagreement with model predictions (Limber 1958b). At the same time, Limber warned that measuring stellar radii would lead to more robust comparisons with stellar evolution models. To this end, he suggested further advances in theory that may be required, including a larger nuclear reaction network to include the burning of lithium and deuterium, departures from hydrostatic equilibrium, "violent atmospheric activity" now associated with magnetic activity, and the influences of rotation and magnetic fields. Yet, despite the obvious complications that could enter into the picture, Limber urged caution (Limber 1958a, pg. 368),

> *We should not attempt to introduce these added complications unless and until the simpler models can no longer account for the observations.*

Building on the framework laid out Limber (1958b) and following the suggestion that improvements to simpler models be explored thoroughly, the next significant advancements in low-mass stellar evolution theory came with the introduction of the mixing length theory (MLT) of convection for stellar models (Böhm-Vitense 1958; Henyey et al. 1965) and improved numerical schemes for solving the set of stellar structure equations (Henyey et al. 1964). These advances permitted more detailed stellar models to be constructed. Several groups created sets of low-mass stellar evolution models that include the aforementioned advances, as well as more detailed stellar atmosphere calculations following a gray $T(\tau)$ construction (e.g., Copeland et al. 1970; Hoxie 1970). These calculations included treatment of various atomic ionization states and molecular dissociation on the EOS and used improved opacity data for bound-free and free-free absorption.

Stellar model calculations were found to be in good agreement with the observational mass-luminosity relationship, but model radii were found to under-predict the radii of stars by up to 40% below about 0.8 $M_\odot$ (Hoxie 1970). Solace was found by noting that observational errors were still dominant, masking any disagreements (Hoxie 1973). However, models also failed to fit the observationally determined properties of YY Gem within 3σ of the observed radius uncertainties, a fact that was not explicitly mentioned. Instead, YY Gem was used to highlight observational uncertainties in the single star effective temperature scale, justifying the large uncertainties in the empirical mass-radius relationship. Nevertheless, potential deficiencies in the mass-radius, and to a lesser extent the mass-luminosity planes motivated Hoxie to identify the largest sources of uncertainty in low-mass stellar models. Two specific pieces of physics were the inclusion of non-ideal effects in the EOS, which would likely affect the stellar radius predictions, and development of more sophisticated boundary conditions, such as non-gray model atmospheres, which would have the largest effect on the stellar $T_{\rm eff}$ and luminosity (Hoxie 1970, 1973).

The next twenty years saw a significant increase in the number of well-characterized EBs (e.g., Popper 1984). Unfortunately, only one low-mass EB was added to the list with

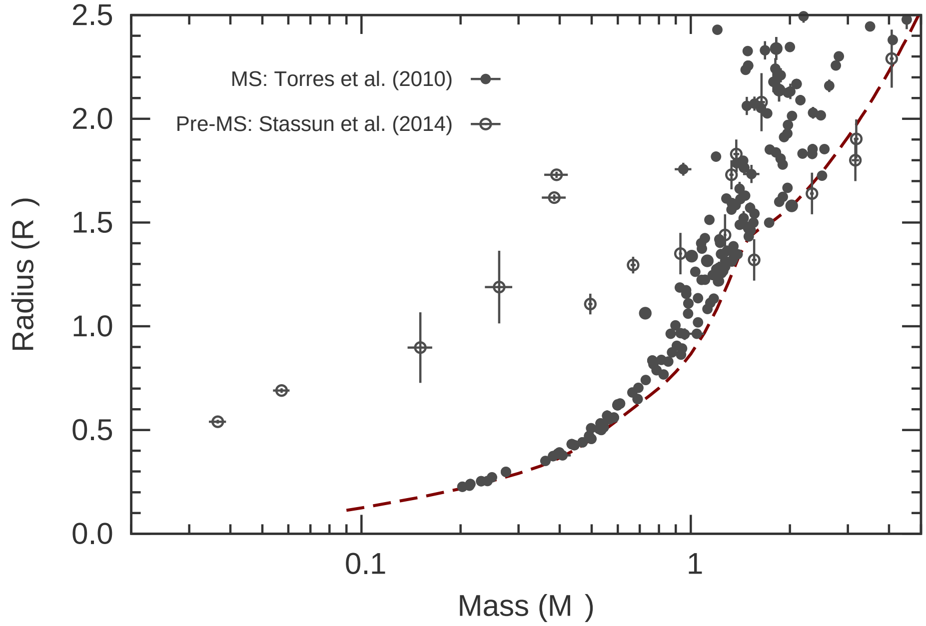

Figure 1: Mass–radius diagram for known eclipsing binaries with masses and radii determined to better than 3% on the MS (filled points; Torres et al. 2010; Kraus et al. 2011; Feiden & Chaboyer 2012a) and 5% on the pre-MS (open points; Stassun et al. 2014). The theoretical ZAMS from Dartmouth stellar evolution models is shown as a dashed line.

YY Gem. That particular system would, however, become just as crucial of a system as YY Gem for comparisons with stellar evolution theory. CM Draconis was characterized by Lacy (1977) and was found to contain two very-low-mass stars with masses similar to those in Krüger 60, meaning they were very likely fully convective throughout their interior (Limber 1958b), although this was not immediately recognized. The stars in CM Dra also appeared to lie well above the theoretical zero-age-main-sequence (ZAMS) for Population I objects in a mass-radius diagram, but this was attributed to the fact that CM Dra may be a Population II object (Lacy 1977).

The surge of well-characterized EBs during these intervening years were compiled by Andersen (1991). In that review, systems were required to have component masses and radii measured with better than 2% precision so that they might act as strong tests of stellar evolution theory. However, there was only one low-mass system in that collection: YY Gem (Leung & Schneider 1978; Andersen 1991). CM Dra's components had masses determined to 4% and radii to 3% (Lacy 1977) and therefore did not merit inclusion in Andersen's compilation.

Low-mass stellar models made their next big leap with the Lyon stellar models (Baraffe et al. 1995). What set their models apart from other groups developing models concurrently (e.g., Dorman et al. 1989; Burrows et al. 1993) was inclusion of an advanced EOS designed specifically to accurately model the cool, dense plasma characteristic of low-mass stars (Saumon et al. 1995) and adoption of non-gray model atmospheres (Allard & Hauschildt 1995) to define surface boundary conditions for their interior models. The latter feature, in particular, allowed their models to accurately treat energy transfer in optically thin regions of the stellar atmosphere, where convection and radiation both contribute significantly to the overall energy flux (Dorman et al. 1989; Burrows et al. 1993; Allard & Hauschildt 1995; Baraffe et al. 1995). Naturally, one of the first tests of their models was comparing model predictions to the properties of YY Gem and CM Dra (Chabrier & Baraffe 1995). Initial results were encouraging, showing that their models were able to reproduce the observational properties of both systems. However, revisions to the fundamental properties of YY Gem and CM Dra would again show models unable to accurately predict stellar properties (Metcalfe et al. 1996; Torres & Ribas 2002). Nevertheless, the physics advances implemented in the Lyon models (Chabrier & Baraffe 1997; Baraffe et al. 1998) remain state of the art, making them still highly relevant.

3 State of the art

In the years since Andersen (1991), there was another explosion in the number of well-characterized EBs, especially in the low–mass regime. To synthesize the wealth of observational data, an updated compilation of EBs with precisely known properties was published by Torres et al. (2010). That review contains 95 systems, but only five systems possess at least one component mass below 0.8 $M_\odot$, and only one system contains a star below the fully convective boundary. The lack of low-mass systems was relieved in subsequent years thanks largely to photometric surveys searching for exoplanets and variable stars, such as *Kepler* (Carter et al. 2011; Doyle et al. 2011), MOTESS-GNAT (Kraus et al. 2011), MEarth (Irwin et al. 2009, 2011), All Sky Automated Survey (ASAS; Hełminiak & Konacki 2011; Hełminiak et al. 2011, 2012, 2014), Super Wide Angle Search for Planets (SuperWASP; Triaud et al. 2013; Gómez Maqueo Chew et al. 2014), and HATNet (Zhou et al. 2014). Figure 1 shows a subset of known EBs (see John Southworth's DEBCat[1] for a complete listing) whose masses and radii are determined with precisions better than 3% and 5% for main sequence (MS) and pre-MS EBs, respectively.

[1] http://www.astro.keele.ac.uk/ jkt/debcat/

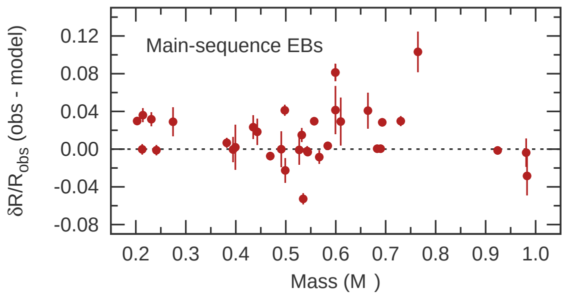

Figure 2: Stellar evolution models fail to reproduce the properties of low-mass main-sequence stars in eclipsing binaries (adapted from Feiden & Chaboyer 2012a).

Standard stellar evolution models of low-mass stars employ similar physics to the Lyon series: surface boundary conditions defined by non-gray model atmospheres, modern radiative opacity computations, and an EOS that accounts for partial electron degeneracy, Coulomb interactions, and other non-ideal effects. The rapid influx of high quality data and high precision mass and radius measurements permits a more reliable evaluation of stellar evolution model accuracy. Such evaluations indicate that the problems identified by Hoxie (1973) still remain, though at a lower level than originally suggested. Canonically, model radii are quoted to underpredict observed values by between 5% and 10% (Ribas 2006). These values can be decreased to below 5%, if potential metallicity and age variations are taken into account (Feiden & Chaboyer 2012a), as shown in Figure 2. However, it is worth noting that the most significant outliers are also the most well-characterized systems, including YY Gem (at 0.6 $M_\odot$) and CM Dra (near 0.23 $M_\odot$).

At the same time, it is clear that problems facing stellar evolution theory below 0.8 $M_\odot$ are not unique to this mass regime. Williams (this volume), for example, illustrates numerous problems that exist with high mass models. Perhaps more worrying, is that studies of EBs with solar-like components reveal that models in the solar-mass regime fail to reproduce observations (Popper 1997; Clausen et al. 2009; Vos et al. 2012). Stellar evolution theorists have previously comforted themselves knowing that, while there are problems in various locations of the HR Diagram, models of the Sun and solar-like stars are accurate. If stars in EBs are representative of the single star population, these errors can have a profound impact on interpretation of asteroseismic data, on characterizations of exoplanet host stars, and potentially on isochrone fitting to globular and open cluster data. Given that modern stellar evolution models show systematic disagreements with EB data, what possible physics may be incomplete or entirely missing from stellar models?

3.1 Observational evidence

The first clue was presented by Mullan & MacDonald (2001), who noted that low-mass EBs showed similar departures from standard stellar model isochrones as single active M-dwarfs, a conclusion supported by additional comparisons of EBs to the single star population (Morales et al. 2008). This lead Mullan & MacDonald (2001) to the hypothesize that magnetic activity may be the culprit for the observed model–observation disagreements. Their hypothesis was supported by the fact that most EBs had short orbital periods, typically less than three days (due to observational biases; Ribas 2006). Nonetheless, stars in these EBs are expected to have rotational periods synchronized with the orbital period, meaning these stars are rapidly rotating (Zahn 1977). Rapid rotation then drives strong magnetic fields through a hydrodynamic dynamo.

Further evidence in support of the magnetic hypothesis was provided when it was shown that radius discrepancies between models and observations of low-mass EBs correlate with magnetic activity in the form of coronal X-ray emission (López-Morales 2007). More active stars, identified as having a higher ratio of X-ray luminosity to bolometric luminosity, were found to show larger disagreements with 1 Gyr, solar metallicity Lyon models. In that same study, there appeared to be no correlation between stellar metallicity and radius deviations for EB systems, a trend that was apparent for single stars and was proposed as an alternative hypothesis to magnetic activity (Berger et al. 2006; López-Morales 2007). This is still some of the strongest evidence in support of the magnetic hypothesis.

A second approach to assessing the influence of magnetic activity is to directly compare model-observation radius disagreements with stellar rotation. Since activity is theorized to increase with increasing stellar rotational velocity, one expects to see shorter period binaries display larger radius deviations, while longer period binaries should show better agreement with model predictions. Kraus et al. (2011) found that short period binaries, with orbital periods less than 1 day, displayed a significantly higher level of radius inflation compared to model predictions than those with periods greater than 1 day. This was supported, in part, by a later study that found a significant break in the level of radius inflation at 1.5 days (Feiden & Chaboyer 2012a). However, no significant difference was found for EBs with orbital periods of 1 or 2 days, suggesting the difference may be due to low number statistics, particularly for systems with orbital periods greater than 3 days. Even accounting for the potential influence of rotation on convection, through the use of the Rossby number,[2] no significant correlation between radius deviations and rotation is observed (Feiden & Chaboyer 2012a). Curiously, Feiden & Chaboyer (2012a) still uncover a correlation with levels of coronal X-ray emission, similar to López-Morales (2007). However, their correlation is neither as clear nor as strong as in López-Morales (2007), and it is observed to be driven largely by two data points, including YY Gem.

Presently, there appears to be no clear observational consensus identifying either magnetic activity or rotation as drivers of the observed radius discrepancies. A few binary systems with orbital periods greater than 18 days are now known whose stars show significant radius disagreements with stellar models (see, e.g., Feiden & Chaboyer 2013a). This does not immediately rule out magnetic activity and/or rotation as a culprit, but certainly adds a layer of complexity to problem. These stars in long period EBs, which show radius disagreement, tend to have masses placing them below the fully convective boundary, perhaps suggesting rotation and magnetic fields are driving errors for models of stars with radiative cores and convective outer envelopes and that models of fully convective stars are plagued by other errors. This evidence has, in any case, inspired theoretical investi-

[2] Ro = $P_{\rm rot}/\tau_{\rm conv}$, the ratio of the convective turnover time to the rotational period

gations into the effects of magnetic fields on low-mass stellar structure.

3.2 Theoretical advancements

The theoretical basis behind suspecting magnetic fields is that strong magnetic fields can suppress the flow of energy across a given surface within a star (e.g., Thompson 1951; Gough & Tayler 1966). Since stars must maintain a given flux at their surface, commensurate with the energy production rate in their core, the inhibition of the outward transfer of flux at any point within the star will force the star to readjust to compensate for the suppressed flux. This compensation is expected to take the form of an inflated radius and decreased $T_{\rm eff}$. The same physical argument applies to both strong magnetic fields globally suppressing convection (e.g., Mullan & MacDonald 2001) or the blocking of radiative flux near the surface by starspots covering some fraction of the stellar surface (Spruit 1982a).

3.2.1 Magnetic suppression of convection

Stellar evolution models that include parametrized descriptions of the influence of global magnetic fields have been constructed by Lydon & Sofia (1995), Ventura et al. (1998), D'Antona et al. (2000), Mullan & MacDonald (2001), and more recently by Feiden & Chaboyer (2012b). Investigation of the effects on low-mass stellar properties in the context of model disagreements with fundamental stellar properties were carried out with the models by Mullan & MacDonald (2001) and Feiden & Chaboyer (2012b), the latter of which uses the prescription derived by Lydon & Sofia (1995). Results from both groups suggest that magnetic suppression of convection is able to reconcile model radius predictions with observations of low-mass benchmark EBs, most notably YY Gem (Feiden & Chaboyer 2013a; MacDonald & Mullan 2014) and CM Dra (MacDonald & Mullan 2012; Feiden & Chaboyer 2014).

In stars with radiative cores and convective envelopes, such as YY Gem, quantitative predictions of surface magnetic field strengths necessary to reconcile model radii appear consistent with empirical data. Model magnetic field strengths range from a few hundred gauss up to a few kilogauss. Direct measurements of magnetic field strengths on active, early M-dwarfs confirm these predictions are realistic (Reiners 2012), as do estimates of magnetic field strengths from indirect indictors, such as chromospheric Ca II emission and coronal X-ray emission (Feiden & Chaboyer 2013a; MacDonald & Mullan 2014). Interior magnetic field strengths predicted by models ($\sim 10 - 100$ kG) also appear consistent with expectations from more realistic magnetohydrodynamic simulations of stellar magnetic fields (Brown et al. 2010). However, models of stars with radiative cores are relatively insensitive to deep interior magnetic field strengths, as the influence of magnetic fields in the superadiabatic layer near the stellar surface is of greater consequence (D'Antona et al. 2000). The broad consistency of model magnetic field strengths with empirical data suggests that magnetic models may be capturing relevant physics for these types of stars, despite simplified magnetic field prescriptions.

The situation for models of fully convective stars, typified by CM Dra, is more complex. Surface magnetic field strengths predicted by models span a similar order of magnitude, anywhere between about 0.5 kG to 5.0 kG (MacDonald & Mullan 2012; Feiden & Chaboyer 2014). These values are reasonable when compared to average surface magnetic field strengths of mid-to-late field M-dwarfs ($\sim$ 3 kG; e.g., Reiners & Basri 2007). Debate about whether magnetic fields are actively inflating fully convective stars largely focuses on predicted interior magnetic field strengths (Chabrier et al. 2007; MacDonald & Mullan 2012; Feiden & Chaboyer 2014). Unlike stars with radiative cores, models of fully convective stars are less sensitive to the strength of the magnetic field in the near-surface layers and appear to be relatively more influenced by interior magnetic field strengths (Mullan & MacDonald 2001; MacDonald & Mullan 2012; Feiden & Chaboyer 2014). Interior magnetic field strengths typically need to be in excess of 1.0 MG. Interested readers are encouraged to consult the aforementioned references for a full account of the arguments for and against the appearance and maintenance of megagauss magnetic fields in fully convective stars. It is likely that stronger constraints from magnetohydrodynamical simulations are needed to further the debate. Nevertheless, there is some concern over the magnitude of interior magnetic field strengths and, thus, global magnetic fields as a solution for the mass-radius disagreements between models and observations.

3.2.2 Magnetic activity – starspots

Alternatively, it may not be global magnetic fields inhibiting convective flows that suppresses flux and forces stars to inflate, but intense local concentrations of magnetic fields producing starspots on the stellar photosphere. Magnetically active stars show modulations in their lightcurves due to the presence of dark spots.[3] Dark spots reduce the radiative output from the stellar surface by trapping excess energy at their base (Spruit 1982b). Depending on the lifetime of a given spot and the efficiency at which the trapped energy can be redistributed, a star may grow larger and cooler in response to the presence of spots (Spruit 1982a).

It was shown that, in the case of low-mass EBs, effects due to spots may be more significant in driving structural changes in a stellar model than effects from global inhibition of convection by magnetic fields (Chabrier et al. 2007). This is of particular consequence for models of fully convective stars, like CM Dra, where global inhibition of convection may not be sufficient, at least with realistic magnetic field strengths (Chabrier et al. 2007; Morales et al. 2010). However, starspots may have an additional influence on studies of EBs. The presence of starspots can bias radius measurements from EB lightcurves (largely through biasing of the measured radius sum) toward larger radii by up to 6%, depending on the properties of spots (Morales et al. 2010; Windmiller et al. 2010). For systems like YY Gem and CM Dra, Morales et al. (2010) estimated that the stellar radii may be overestimated by about 3%. It is not possible to necessarily attribute all of the observed errors to this bias, but assuming these stars have smaller radii assists theoretical models of radius inflation through magneto-convection or starspot flux suppression by lowering the required amount of radius

[3] There is the distinct possibility that lightcurve modulations are caused by bright spots, but it has been argued that this is unlikely (Berdyugina 2005).

inflation (Morales et al. 2010; MacDonald & Mullan 2012).

Critical to the idea that spots bias radius measurements is that spots are preferentially located near the rotational poles and cover a significant fraction of the stellar surface (between 40 – 60%). Suppression of flux by spots at the surface is not sensitive to distribution, but is highly dependent on surface coverage and spot temperature contrasts. Presently, there is no strong empirical evidence for polar spots on either rapidly rotating low-mass stars or fully convective stars (see Section 4.3 in Feiden & Chaboyer 2014, for an in-depth discussion). In fact, there is evidence that spots must be randomly distributed across the surface of fully convective stars to produce the diversity of lightcurve morphologies for stars in the young open cluster NGC 2516 (Jackson & Jeffries 2013). This leads to the possibility that fully convective stars have random distributions, but large filling factors (total surface coverage). Observations of FeH molecular band features in spectra M-dwarfs reveal that M-dwarfs are probably covered nearly everywhere by 1 kG magnetic fields, with small patches of intense 5 kG – 8 kG magnetic fields, averaging to about 3 kG over the surface (Shulyak et al. 2011), supporting this idea. However, this assumes a one-to-one correlation between the presence of 1 kG magnetic fields and the appearance of spots. Investigations of solar magnetic fields reveal that 1 kG magnetic fields may not be sufficient to inhibit convective flows (Mathew et al. 2004). Convective flows in low-mass stars likely react differently to the presence of magnetic fields than convective flows in the Sun, but it raises the question of whether starspot properties required by stellar models are realistic.

4 Exploring alternative solutions

While magnetic fields and activity are able to provide a solution to the mass–radius problem with low–mass stars, questions remain about the reality of stellar evolution model predictions. These questions will undoubtedly be answered by future observations. However, perhaps an equally valid epistemological approach is to explore alternative theoretical solutions with the aim of removing the need to invoke magnetic fields, or with the aim of ruling out the other alternatives and thus bolstering arguments in favor of the magnetic hypothesis. Recalling discussions in Section 2, this is the approach advocated by Limber (1958a).

4.1 Convection

One alternative to the magnetic hypothesis is that convection in low-mass stars is significantly less efficient than it is in more solar-like stars. This argument has been explored previously, but investigations have largely focused on individual systems. Suggestions for why convective properties may be different among individual EBs that show radius disagreements with models include magnetic fields (Cox et al. 1981), rotation (Coriolis force acting on convective flows; Chabrier et al. 2007), and intrinsic differences due explicitly to stellar properties (i.e., mass, metallicity, $T_{\rm eff}$; Lastennet et al. 2003). While it is important to demonstrate that manipulating convective properties—here largely the convective MLT parameter $\alpha_{\rm MLT}$—provides relief to noted model–observation disagreements, it makes identifying physical explanations for those manipulations difficult. Instead, statistical properties from a sample of EBs provides an opportunity to reveal meaningful trends that betray physics associated with changes in stellar convective properties.

Such a study was recently performed for solar-like EBs (Fernandes et al. 2012), where $\alpha_{\rm MLT}$ was manipulated to provide the best agreement between models and EBs. Results show that $\alpha_{\rm MLT}$ does not appear to be mass dependent, as stars of equal mass from different EB systems require different values for $\alpha_{\rm MLT}$. Instead, Fernandes et al. (2012) demonstrate that $\alpha_{\rm MLT}$ correlates with $v \sin i$, where faster rotating stars require lower $\alpha_{\rm MLT}$ values. While the results are tantalizing, comparison of models against solar-like EBs introduces errors due to unknown stellar ages and compositions (both helium abundance Y and metal abundance Z), which must be simultaneously fit. Although stars in a given EB must lie along the same isochrone, which isochrone is assumed correct can decidedly alter modeling conclusions. Extending this type of study to low-mass EBs would, however, help relieve modeling uncertainties associated with unknown stellar ages and compositions, as low-mass stellar models are relatively less sensitive to assumptions about these parameters. Having at least some constraint on stellar metallicity can drastically improve the quality of model comparisons, so observers are anyway encouraged to put forth effort to measure stellar metal content.

It is quite remarkable that, through a systematic comparison of the low-mass EB population to stellar models, it may be possible to extract information regarding the hydrodynamic properties of low-mass stars as a function of a range of variables, such as mass, $[M/{\rm H}]$, $T_{\rm eff}$, and $v \sin i$ in a similar manner as is done with asteroseismology (Bonaca et al. 2012). This knowledge would lead to significant improvements in the treatment of convection in stellar evolution models and would represent yet another facet of stellar physics advanced by studies of EBs.

4.2 Heavy element composition

Stellar metallicity plays only a minor role in modern discussions of the low-mass EB mass-radius problem. However, it is not clear, at least to the author, that metallicity effects are not partially responsible for the observed disagreements. Although López-Morales (2007) failed to identify a correlation between metallicity and radius inflation among low-mass stars in binaries, the lack of a trend was largely driven by the points associated with CM Dra. Revisions to the fundamental properties of CM Dra (Morales et al. 2009) and recent estimates of its metallicity (Terrien et al. 2012; Kuznetsov et al. 2012) have led to a shift in the location of CM Dra in the metallicty–radius inflation diagram, revealing circumstantial evidence that stellar metallicity may be related to observed radius errors among fully convective stars (Feiden & Chaboyer 2013b).

However, correlations with metallicity among fully convective stars in EBs does not follow the same pattern as was observed for single stars (Berger et al. 2006). Instead of more metal rich stars showing larger radius inflation, radius errors among fully convective stars in EBs appear to negatively correlated with stellar metallicity, with more metal poor stars displaying greater radius errors. This may point toward errors in the model EOS or opacities. However, the sample is small, with only six stars in four systems, and is largely driven by the location of CM Dra in the diagram. The notion is bolstered slightly by the fact that it does not matter what metallicity is adopted for CM Dra, it still falls along

the observed negative correlation. It should be noted that a recent investigation by Zhou et al. (2014) claims to find no trend of radius inflation with metallicity among fully convective stars in EBs. However, their most precisely measured star, and their most metal-poor ($[M/\mathrm{H}] = -0.6$ dex), falls along the relation suggested by Feiden & Chaboyer (2013b). Confirmation of the properties of other stars in their sample is required. As the number of fully convective stars in EBs with metallicity estimates is increased, it will become clear whether hints of the trend are spurious or not.

4.3 Helium abundance

An interesting possibility is to use low-mass EBs to investigate stellar helium abundances (Limber 1958b). Helium abundances in low-mass stars cannot be measured directly, and therefore specification of Y in stellar models relies on prior assumptions regarding the relationship between Y and Z. The significant role that helium plays in governing the structure and evolution of low-mass stars, coupled with our ignorance of Y in low-mass stars, makes it a noteworthy suspect in our attempts to solve the mass-radius problem (Valcarce et al. 2013). Although observational confirmation of model predicted Y values cannot be obtained, consistency with other measures of local helium abundances in the solar neighborhood can be sought.

As with studies of adjusting α_{MLT}, previous investigations largely focused on individual systems, such as CM Dra (Paczynski & Sienkiewicz 1984; Metcalfe et al. 1996) and UV Psc (Lastennet et al. 2003). Populations of solar-like EBs were used to probe the helium enrichment as a function of metallicity in the solar neighborhood (Ribas et al. 2000; Fernandes et al. 2012). It is usually assumed that helium abundance is linearly proportional to metallicity, such thats

$$Y(Z) = Y_{\mathrm{P}} + \frac{\Delta Y}{\Delta Z} \cdot Z, \tag{1}$$

where Y_{P} is the primordial helium abundance. Solar-like EBs suggest that the slope of the relation is $\Delta Y/\Delta Z = 2 \pm 1$ (Ribas et al. 2000; Fernandes et al. 2012), consistent with estimates from single K-dwarfs in the solar neighborhood (e.g., Casagrande et al. 2007). However, the inferred primordial helium abundance from these studies is well below the primordial abundance estimated from Big Bang Nucleosynthesis ($Y_P \approx 0.225$ compared to $Y_{P,\,\mathrm{BBN}} \approx 0.249$; Peimbert et al. 2007).

Inferring helium abundances involves simultaneously fitting multiple stellar model parameters (i.e., $\alpha_{\mathrm{MLT}}, Y, Z$, age) to provide the best possible agreement between model predictions and observed properties of stars in EBs. It was mentioned in Section 4.1 that these parameters have a strong impact on stellar model calculations of solar-like stars, introducing degeneracies in the optimization problem. Presently, very few EBs have metallicity estimates, making it difficult to add observational priors in the optimization scheme. Metallicity estimates would help tremendously, particularly for solar-like stars. At the same time, with the recent increase in the number of well-characterized low-mass EBs, it is now possible to use low-mass EBs to constrain $Y(Z)$. Models are considerably less sensitive to fit parameters, particularly age, as compared to solar-like stars. Metallicities are still a concern, but methods of estimating bulk metallicities for low-mass stars are showing promise (Rojas-Ayala et al. 2012; Mann et al. 2013). Assuming how helium abundances scales with metallicity is a crucial in stellar evolution modeling, and studies of low-mass EBs provide the best chance to reveal this relation.

4.4 Probing internal structure

Perhaps the ultimate test of stellar evolution theory that EBs can provide is a direct inference of a star's internal density structure through the measurement of apsidal motion in EBs with eccentric orbits. This is very fitting for this occasion, where we gather to remember the contributions to binary star science by Zdeněk Kopal. Observing the precision of an EB orbit's periastron with sufficient accuracy and precision yields an average interior structure constant, $\overline{k_2}$ of the two component stars. For any individual star, k_2 quantifies the object's central mass concentration (Kopal 1978). Objects that are very centrally condensed, with tenuous outer layers and dense cores compared to the average density (i.e., high mass stars) are described by lower k_2 values than objects that have a more even distribution of mass (i.e., M-dwarfs).

The fact that observations only reveal the component averaged k_2 value means that equal mass binaries are preferable. Fortunately, low-mass binaries are more likely to be of equal mass (Bate, this volume). However, low-mass stars have a lower fraction of binarity and typically form with smaller semi-major axes (see Bate, this volume), meaning they will tend to rapidly circularize from tidal interactions (Zahn 1977). All considered, the properties of low-mass binaries makes the chances of finding suitable systems for deriving k_2 quite low.

Despite this, if apsidal motion in a low-mass binary is accurately measured, we will learn a great deal about the accuracy of stellar models. Primarily, we will learn whether models predict accurate density profiles for the deep interiors of low-mass stars. The apsidal motion constant is largely determined by the deep interior structure, with smaller influences from the near surface layers, where the density in individual layers is much less than the average density of the star. If models accurately predict k_2 for stars where models do not predict accurate fundamental properties (radius, T_{eff}), then the problems can be isolated to the near surface layers. In contrast, if the deeper layers in models are found to be inadequately described, this points towards a different set of physics, such as opacities and the EOS. At least one low-mass binary gives us this opportunity, KOI-126 (Carter et al. 2011), where the apsidal motion constants may be determined for two fully convective stars with a precision of 1%.

5 Conclusion

Our current understanding of modeling errors are very much like the early view on M-dwarf stars (Russell 1917, commenting on Krüger 60),

> *It is obvious that these stars exhibit every characteristic which might be supposed typical of bodies at the very end of their evolutionary history, and on the verge of extinction.*

At the time, this view of M-dwarfs was well justified, but nonetheless it now appears rather strange, as M-dwarfs are thought to live for hundreds of billions of years, meaning they are actually at the very beginning of their evolutionary history. Indeed, it is very easy to grow confident that

observed errors between models and stars in EBs are themselves becoming extinct. However, as history of comparing models to theory has continually demonstrated, more often than not, errors persist even with the most sophisticated models. It is entirely likely that, as M-dwarfs are actually at the very beginning of their lives, we are only at the beginning of our journey of reconciling errors between stellar evolution theory and observations. It is clear, though, that well-characterized EBs, like YY Gem, will continue to play critical roles in advancing stellar evolution theory.

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